Synchronization, Chaos & Quantum Chaos

Thursday, Sept. 3: 11:00 – 13:00

Session 2: Hilbert Hall

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Novel Quantum-Chaotic Ratchet Effects: Full Symmetry and Planck Maximal Uniformity

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Classical Hamiltonian ratchets may exhibit a directed chaotic current only for an asymmetric system with a mixed phase space, featuring “transporting” stability islands. Such a current can never arise if just one of the following three conditions is satisfied: (a) The system is fully chaotic; (b) The system is completely symmetric; (c) The initial ensemble is uniform in phase space. Already several works have shown that an asymmetric quantum Hamiltonian system with an unbiased force can exhibit significant ratchet effects also when its classical counterpart is *fully chaotic* [condition (a)].

In our work [1], we have presented a general exact theory of quantum ratchets for the kicked particle subjected to a linear potential (e.g., gravity) under quantum-resonance conditions (strong quantum regime) in the “free-falling" frame. It was found that significant quantum-ratchet effects emerge under full-chaos conditions also when both the kicking potential and the initial wave packet are *completely symmetric* [condition (b)], see Fig. 1.1.

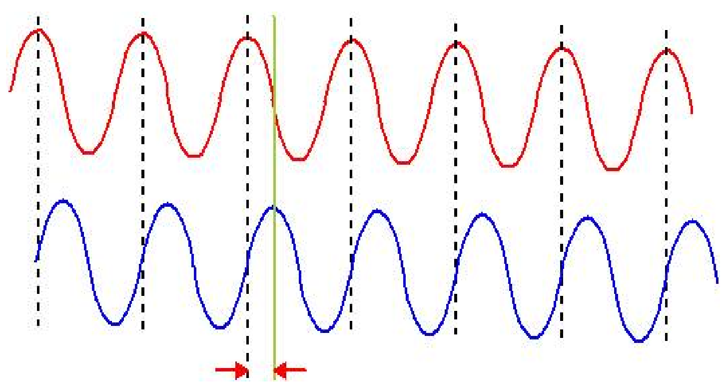

Fig. 1.1: *Completely symmetric* (cosine) kicking potential (blue line) and initial wave packet (red line). A *purely quantum* ratchet acceleration arises just from the noncoincidence of the symmetry centers of these two entities.

Recently [2], a simple case of these “symmetric” effects in the absence of a linear potential was experimentally realized using atom-optics techniques with Bose-Einstein condensates (BECs) kicked by an optical potential. A good agreement was found between the experimental results and the theory [1], properly modified to take into account the small but finite initial momentum width of the BEC.

A first study of the semiclassical regime of full-chaos quantum ratchets was presented in our very recent work [3]. Since the quantum-ratchet effect is quite sensitive to the initial state [1] and since a uniform classical ensemble in phase space carries no current [condition (c)], this study was performed using a natural *global* approach, taking into account *all* the initial states that are uniform in phase space with the *maximal possible* resolution of one Planck cell. Our main result in a strong-chaos regime is that the distribution of quantum currents I over all these states is a *symmetry-independent* Gaussian with mean $\langle I \rangle = 0$ and variance $\langle I^2 \rangle \approx D\hbar^2/(2\pi^2)$, where D is the classical chaotic-diffusion coefficient and $\hbar$ is a scaled Planck constant tending to zero in the classical limit; see Fig. 1.2.

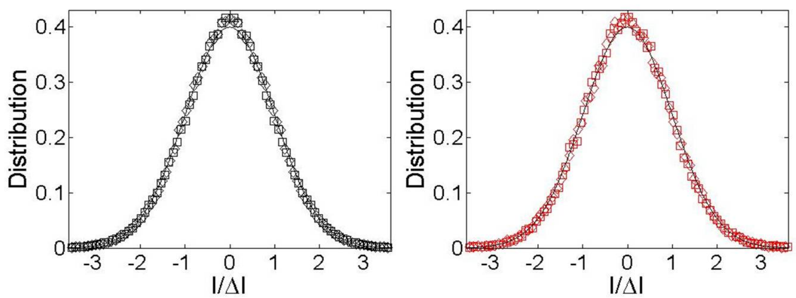

Fig. 1.2: Left: Distributions of normalized quantum currents $I/\Delta I$ ($\Delta I = \sqrt{\langle I^2 \rangle}$) for $\hbar = 2\pi/121$ and two values of the kicking strength (squares and diamonds) in the case of a *completely symmetric* chaotic system; the solid line is a variance-1 Gaussian. Right: Similar to left figure but for a *strongly asymmetric* chaotic system.

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2

From Fragile to Robust Pseudo-Hermitian Phase in $\mathcal{PT}$-Symmetric Lattices with Localized Eigenmodes

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Hamiltonians exhibiting parity-time ($\mathcal{PT}$) symmetry have been the subject of a rather intense research activity during the last few years [1-9]. This interest was motivated by various areas of physics, ranging from quantum field theories, and mathematical physics to solid state and classical optics. A surprising result that was pointed out in some of these investigations was the possibility that non-hermitian $\mathcal{PT}$ symmetric Hamiltonians can have a purely real eigenvalue spectrum. At the same time, it was found that these systems exhibit a spontaneous $\mathcal{PT}$ symmetry breaking during which the eigenvalue spectrum undergo a transition [1] from an entirely real spectrum (the $\mathcal{PT}$-symmetric phase) towards a partially, or completely complex spectrum (the phase with broken $\mathcal{PT}$-symmetry). Usually this transition is controlled by a parameter of the Hamiltonian.

We study the effect of localized modes in lattices of size N with parity-time ($\mathcal{PT}$) symmetric potentials. Such modes are arranged in pairs of quasi-degenerate levels with splitting $\delta \sim \exp^{-N/\xi}$ where ξ is their localization length. The level "evolution" with respect to the $\mathcal{PT}$ breaking parameter γ shows a cascade of bifurcations during which a pair of real levels becomes complex. The spontaneous $\mathcal{PT}$ symmetry breaking occurs at $\gamma_{\mathcal{PT}} \sim \min\{\delta\}$, thus resulting in an exponentially narrow exact $\mathcal{PT}$ phase. As N/ξ decreases, $\gamma_{\mathcal{PT}}$ scales as $1/N^2$ while its distribution changes from log-normal to semi -Gaussian [10]. We propose a way to preserve the robustness of the pseudo-hermitian phase by using dimer lattices that posses a generalized $\mathcal{PT}$ symmetry [11]. Our theory can be tested in the frame of $\mathcal{PT}$-optical lattices.

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3

Avalanches of Bose-Einstein Condensates in Leaking Optical Lattices

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We study the decay of an atomic BEC population $N(\tau)$ from the leaking boundaries of an optical lattice (OL). For a rescaled interatomic interaction strength $\Lambda > \Lambda_b$, discrete breathers (DBs) are created that prevent the atoms from reaching the leaking boundaries. Collisions of other lattice excitations with the outermost DBs result in avalanches, i.e. steps in $N(\tau)$, which for a whole range of $\Lambda-$values follow a scale-free distribution $P(J = \delta N) \sim 1/J^{\alpha}$. A theoretical analysis of the mixed phase-space of the system indicates that $1 < \alpha < 3$, in agreement with our numerical findings.

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Studies of resonances in open microwave cavities by the method of harmonic inversion

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From the measurement of a reflection spectrum of an open microwave cavity the poles of the scattering matrix in the complex plane have been determined [1]. The resonances have been extracted by means of the harmonic inversion method [2]. By this it became possible to resolve the resonances in a regime where the line widths exceed the mean level spacing up to a factor of 10, a value inaccessible in experiments up to now. For example the distributions of line widths were studied and found to be in good agreement with predictions from random matrix theory [3].

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5

Spreading of wave packets in one dimensional disordered chains: Different dynamical regimes

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We present numerical results for the spatiotemporal evolution of a wave packet in quartic Klein-Gordon (KG) and disordered nonlinear Schrödinger (DNLS) chains, having equivalent linear parts. In the absence of nonlinearity all eigenstates are spatially localized with an upper bound on the localization length (Anderson localization). In the presence of nonlinearity we find three different dynamical behaviors depending on the relation of the nonlinear frequency shift δ (which is proportional to the system's nonlinearity) with the average spacing $\overline{\Delta\lambda}$ of eigenfrequencies and the spectrum width Δ ($\overline{\Delta\lambda} < \Delta$) of the linear system. The dynamics for small nonlinearities ($\delta < \overline{\Delta\lambda}$) is characterized by localization as a transient, with subsequent subdiffusion (regime I). For intermediate values of the nonlinearity, such that $\overline{\Delta\lambda} < \delta < \Delta$ the wave packets exhibit immediate subdiffusion (regime II). In this case, the second moment m_2 and the participation number P increase in time following the power laws $m_2 \sim t^{\alpha}$, $P \sim t^{\alpha/2}$. We find $\alpha = 1/3$. Finally, for even higher nonlinearities ($\delta > \Delta$) a large part of the wave packet is selftrapped, while the rest subdiffuses (regime III). In this case P remains practically constant, while $m_2 \sim t^{\alpha}$.

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6

Loss of synchronization in delay-coupled lasers: on bubbling and on-off intermittency

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Semiconductor lasers are of particular interest in the study of chaos synchronisation. However, if two identical lasers are optically coupled over a finite distance, it has been observed that the coupling delay leads to spontaneous symmetry breaking [1]. A passive relay in form of a semitransparent mirror or an active relay in form of a third laser in between the two lasers have been shown to stabilize the isochronous synchronisation solution [2, 3].

We numerically and analytically study this zero-lag chaos synchronisation of two lasers which are delay-coupled via a passive or an active relay. We show that both setups exhibit bubbling [4, 5], i.e., noise-induced desynchronisation, or on-off intermittency depending on the coupling parameters.

In case of a passive relay, the synchronised system behaves like a single laser with feedback. In the coherence collapse (CC) regime the trajectory itinerates among the modes and antimodes (see Fig. 6.1(c)). The modes involved in the chaotic itinerancy are transversally stable (blue circles), while the antimodes are transversally unstable (red squares). Thus, when the trajectory approaches an antimode, noise can lead to desynchronization (see Fig.6.1(a)). The yellow diamonds in Fig. 6.1(c) mark the onset of a desynchronization event, showing that bubbling always occurs in the vicinity of the antimodes, independent of the power.

In the low frequency fluctuation (LFF) regime, in between power dropouts the trajectory switches between different attractor ruins of unstable external cavity modes (ECMs) with a drift towards the maximal gain mode. All ECMs involved in this intensity buildup process are transversally stable and we observe no desynchronization. During the power dropout the trajectory collides with an antimode in a crisis. Again, the vicinity to transversally unstable antimodes, leads to bubbling (see Fig. 6.1(b)). This behaviour has also previously been observed in unidirectionally coupled lasers [6]. With increasing feedback the bubbling occurs less frequently, but it remains present within a physically reasonable range of the coupling strength.

We find similar bubbling behaviour if we replace the passive relay by an active relay, which is identical to the outer lasers (and has the same pump current). However, when we increase the pump current of the middle laser, we do find a transition to a bubbling-free state. In this case the system still itinerates among the compound laser modes, but all the modes involved in the dynamics are indeed transversally stable (see Fig. 6.1(d)).

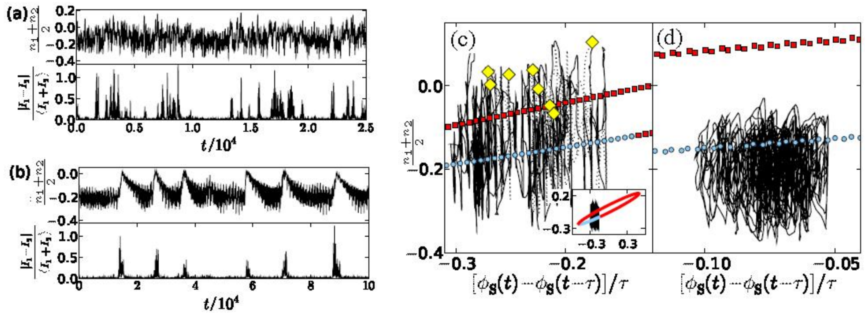

Fig. 6.1: Left: Time series for the passive relay case a) in the CC regime b) in the LFF regime. Right: Projection of the dynamics onto the $(\omega_\mathbf{S}, n_\mathbf{S})$-plane for c) passive relay (CC regime) d) strongly pumped active relay. Blue circles and red squares display transversally stable and unstable modes respectively. Yellow diamonds indicate the onset of desynchronization. The variables n_i, I_i and ϕ_i are the excess carrier density, the intensity and the optical phase of laser i, respectively (index $\mathbf{S}$ denotes a symmetrized variable),

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7

Coexistence of Synchronized and Desynchronized States in Nonlocally Coupled Oscillator Systems

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We study networks of phase-coupled oscillators, based on the Kuramoto model [3, 4], with nonlocal coupling

$$\frac{\partial}{\partial t}\phi(\mathbf{x},t) = \omega + \int_D G(\mathbf{x}-\mathbf{x}')\sin[\phi(\mathbf{x}',t)-\phi(\mathbf{x},t)+\alpha]\,d^2\mathbf{x}, \tag{7.1}$$

where ϕ are the phases of individual oscillators with natural frequency ω located at $\mathbf{x}$ in the domain D. The nonlocal coupling is given by a kernel G such that the coupling strength decays with distance on the domain, e.g. $G \sim \exp(-\kappa|x|)$. Recently, Kuramoto *et al.* discovered that such systems exhibit intriguing states where regions of synchronized and desynchronized oscillators coexist and form a stable pattern [5, 6]. This phenomena has since

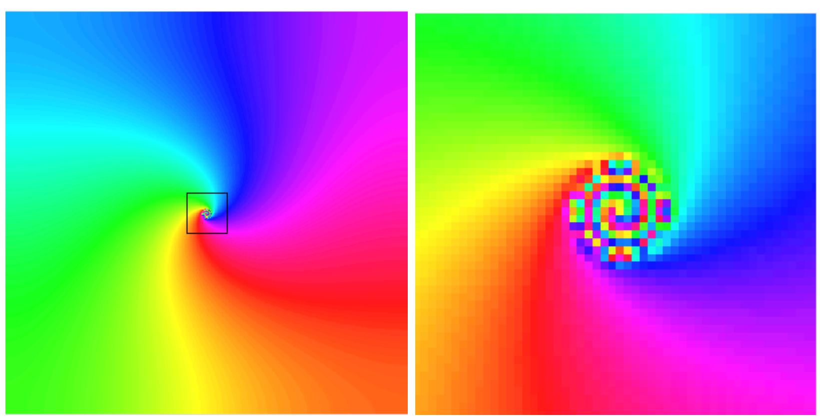

Fig. 7.1: Example of a chimera state on a 2D lattice of oscillators with spiral wave. A closeup of the desynchronized core is shown on the right.

then gained strong interest in the community and been investigated in various settings to better understand its specific nature [1, 2, 7, 11].

We present a study concerning simple networks of oscillator populations with nonlocal coupling. In particular, we find the existence of bistable chimera attractors and study the influence of the underlying network topology on the existence of chimerae [8–10]. A recent breakthrough in the field of coupled oscillators made by Ott and Antonsen [12,13] enables us to perform a complete stability analysis for these types of systems.

Chimera states also occur on 2D lattices of oscillators in the shape of spirals: in this case, the spiral arms form the synchronized region, whereas the typically observed topological singularity in the center is replaced with a zone of desynchronized oscillators, as shown in Fig. 7.1. We demonstrate how to obtain analytical solutions of this system and present a method to calculate the radius of the desynchronized core analytically.

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8

Phase Condensation in Bursting Chaos with Weak Periodic Forcing

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Synchronization phenomena are ubiquitous in biology, e.g. pacemaker cell in the heart, neuronal systems, flashing fireflies, and so on. Recently, those synchronization phenomena can be modeled as a phase synchronization in weakly coupled limit cycle oscillators with a certain phase. Then, phase synchronization in those oscillators has much gathered attention during a few decades [Strogatz(2000)].

Moreover, such kind of phase synchronization can be generalized in chaotic oscillators [Pikovsky et al.(1997)]. At the first stage, chaotic phase synchronization (CPS) has been studied in a chaotic oscillator with one rotation center like the Rössler system. Therefore, there are several ways to define the phase of the chaotic oscillators.

On the other hand, complex systems like biological system have not only a simple rotation but also complex rotations such as neuronal spiking or bursting [Connors and Gutnick(1990)]. However, in contrast to an ordinary CPS for single rotation center, it is not straightforward to define the phase of the non-coherent chaotic oscillations.

Recently, Pereira *et al.* proposed a concept of the localized sets in order to detect phase synchronization in a multi-time-scale neuronal system which has several rotation centers [Pereira et al.(2007)]. Suppose that a chaotically bursting neuron is driven by a periodic force. If there is a phase synchronization between the neuron and the driving force, localized sets on chaotic attractor for the stroboscopic map can be detected.

In this presentation, let us focus on a system with very weak driving force, where localized sets cannot be observed, i.e. the stroboscopic map is not localized. We investigate the neural bursting system proposed by Chay [Chay(1985)] with the weak sinusoidal force $K \sin(\Omega t)$. This bursting system has two separated modes with complex rotation centers, i.e. slow and fast modes which correspond to bursting oscillations and refractory periods. We add the sinusoidal driving force with the corresponding fast mode.

First, in order to characterize the correlation between the dynamics of the system and the driving force, we calculate the time evolution of the ensemble average of N-identical uncoupled bursting systems driven by the same driving force. We investigate the dependence of the variance and the power spectrum of the time evolution on the amplitude of the driving force K.

Next, according to the concept of the localized sets, we define the phase of the driven chaotic dynamics by corresponding phase of the periodic driving force when the chaotic orbits pierce an appropriate Poincaré section in the driven system. Using this phase, we analyze the distribution of the phases of uncoupled ensemble of bursting systems.

Interestingly, we have found a non-trivial phase transition with respect to K in the vicinity of natural frequency of the fast mode for Ω as shown in Fig.8.1 (Left). There is a plateau in the phase transition. Furthermore, in very weak amplitudes of forcing K, we can observe a condensation in the distribution of the phases of the ensemble with increasing K (see Fig. 8.1 (Right)), where no clear phase synchronization is observed in the sense of the localized sets. In fact, we can observe phase synchronization between the ensemble average of the driven systems and the same driving force.

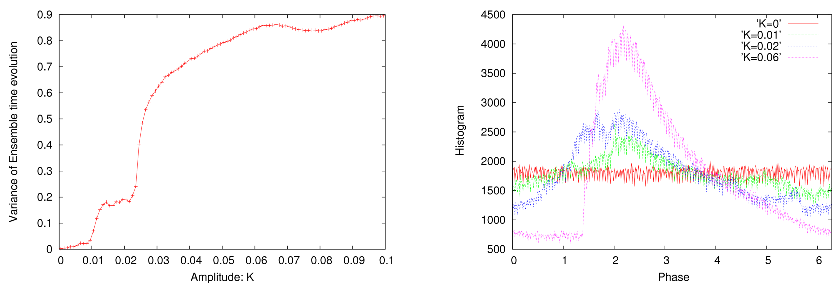

Fig. 8.1: (Left) Variance of the time evolution of ensemble with respect to the amplitude of forcing K. (Right) Histogram of the phases of the ensemble with respect to different K. $K = 0, 0.01, 0.02$, and 0.06.

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Synchronization and Collective Transport in linearly coupled Inertia Ratchets

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Transport phenomena and in particular directed transports of nonlinear non-equilibrium dynamical systems modelled by ratchet systems are ubiquitous in nature. A large number of researches have been devoted to the understanding of the ratchet effect for the one-dimensional ratchet system. In reality, one cannot find an isolated particle system. Thus, one fundamental problem relates to the ratchet transport mechanism when two or more isolated ratchets interact via a specific coupling. In this paper, we consider the dynamics of two elastically coupled inertia ratchets in a perturbed asymmetric potential. We show that the particle-particle interactions could lead to varieties of collective transports ranging from stable on-off intermittent synchronized state to full synchrony. The fully synchronized state is achieved via a transition from on-off intermittent synchronized state accompanied by crisis event in which the particles exhibits stepwise-sliding dynamics typical of the actin-myosin systems in muscles. We show that the dynamics and the corresponding transport mechanism is strongly dependent on the strength, ϵ of the particle-particle interaction and in particular, optimal and enhanced transport could arise as ϵ is progressively increased up to the fully synchronized state.

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