

Biological, Socio-Economic & Evolutionary Dynamics

Wednesday, Sept. 2: 11:00 – 13:00

Session 2: Hilbert Hall

Contents

1 Theoretical modelling of bacterial motor dynamics <i>E. Baresel, and R. Friedrich</i>	3
2 A MRI-based approach to introduce cardiac motion into simulations of electrical excitation <i>S. Fruhmer, S. Bauer, H. Engel, and M. Bär</i>	4
3 Subordinated Langevin Equations for Anomalous Diffusion in External Potentials <i>S. Eule</i>	5
4 Coevolutionary dynamics in well-mixed finite populations: how stochastic stability depends on the process <i>Jens Christian Claussen, and Arne Traulsen</i>	6
5 Quasispecies, quantum mechanics and the survival of the flattest <i>J. Krug, and A. Wolff</i>	7
6 Bivariate time periodic Fokker-Planck model for freeway traffic <i>F. Lenz, A. Riegert, D. Herde, and H. Kantz</i>	9
7 Very noisy Fisher waves <i>O.H. Hallatschek, and K.S. Korolev</i>	11
8 Crime avalanches maybe the effect of unpunished crimes and also of jail “contamination” <i>J.R. Iglesias</i>	12

Theoretical modelling of bacterial motor dynamics

E. Baresel and R. Friedrich

Institute for Theoretical Physics, Westfälische Wilhelms-Universität Münster

As a model of bacterial motors we consider the dynamics of an ensemble of swimming objects which are composed of two rigidly connected point vortices. The single objects are able to propel Fig. 1.1 or to tumble Fig. 1.2 depending on the circulations of the single point vortices. We discuss the collective behaviour for several of these objects by means of numerical calculations.

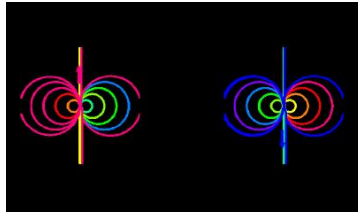


Fig. 1.1: Model of a streamer.

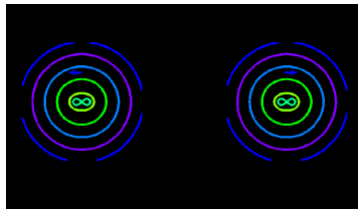


Fig. 1.2: Model of a simple rotator.

A MRI-based approach to introduce cardiac motion into simulations of electrical excitation

S. Fruhner¹, S. Bauer², H. Engel¹, and M. Bär²

¹ Technische Universität Berlin, Germany

² Physikalisch-Technische Bundesanstalt, Germany

The propagation of electric waves controls the cardiac contraction. In simulations realistic heart models often include detailed physiological knowledge about ionic dynamics of cardiac cells and accurately account for anatomical details like fibre orientation or heterogeneity of heart tissue. On the other hand mechanical deformations of the heart during contraction usually is neglected. The feedback between propagating waves of electric activity and cardiac contraction cannot be described by static models but it might be essential for understanding the mechanism of cardiac arrhythmias like tachycardia and fibrillation.

Using magneto-resonance images two-dimensional finite-element meshes have been generated in order to perform simulations of waves of electric activity propagating in a beating human heart. The approach offers the opportunity to calculate the mechanical stresses during cardiac contraction from experimental data without using detailed models on calcium dynamics and stress-activated channels in cardiac myocytes. First results will be presented.

Subordinated Langevin Equations for Anomalous Diffusion in External Potentials

S. Eule

Max-Planck-Institute for Dynamics and Self-Organization, Bunsenstrasse 10,
37073 Göttingen

The role of external forces in systems exhibiting anomalous diffusion is discussed on the basis of the describing Langevin equations. Since there exist different possibilities to include the effect of an external field the concept of *biasing* and *decoupled* external fields is introduced [1]. Complementary to the recently established Langevin equations for anomalous diffusion in a time-dependent external force-field by Magdziarz et al. [2] the Langevin formulation of anomalous diffusion in a decoupled time-dependent force-field is derived. Thereby the mathematical concept of subordination is applied [3].

References

1. S. Eule, R. Friedrich, to appear in EPL (2009).
2. M. Magdziarz, A. Weron, J. Klafter, Phys. Rev. Lett. **101**, 210601 (2008).
3. H. C. Fogedby, Phys. Rev. E **50**, 1657 (1994).

Coevolutionary dynamics in well-mixed finite populations: how stochastic stability depends on the process

Jens Christian Claussen^{1,2} and Arne Traulsen³

¹ Institut für Neuro- und Bioinformatik, Universität zu Lübeck,
Ratzeburger Allee 160 - Geb. 64, 23562 Lübeck, Germany

² Institut für Theoretische Physik und Astrophysik,
Christian-Albrechts-Universität zu Kiel, 24098 Kiel, Germany

³ Max-Planck-Institut für Evolutionsbiologie, August-Thienemann-Str. 2, 24306
Plön, Germany

Coevolutionary dynamics is investigated in chemical catalysis, biological evolution, social and economic systems. The dynamics of these systems can be analyzed within the unifying framework of evolutionary game theory. In [1] we have discussed the derivation of macroscopic (mean-field) equations of motion from microscopic processes with interesting implications on the stability of the coexistence fixed point. Here, we present an analytic approach to assess the stability of cyclic coevolutionary dynamics in finite populations, both for the rock-paper-scissors (RPS) game [2] and for the [3] asymmetric biological battle of the sexes. For the latter, the dependence on the microscopic processes can be formulated more formally, coexistence is stabilized by processes both nonlinear and nonlocal. For the RPS game (which is played within a single population) a similar effect occurs if the game is non zero-sum: if the bank loses (as for *E.coli* bacteria) coexistence is lost, if the bank wins (as for lizards) coexistence is stabilized - without help of spatial niches.

References

1. A. Traulsen, J. C. Claussen, and C. Hauert, Phys. Rev. Lett. 95, 238701 (2005).
2. J.C. Claussen and A. Traulsen, Phys. Rev. Lett 100, 058104 (2008)
3. Jens Christian Claussen, European Physical Journal B 60, 391-399 (2007)

Quasispecies, quantum mechanics and the survival of the flattest

J. Krug and A. Wolff

Institut für Theoretische Physik, Universität zu Köln, Zùlpicher Strasse 77, 50937 Köln, Germany

In current evolutionary theory, the concept of *robustness*, referring to the invariance of the phenotype under perturbations, is of central importance. To be concrete, suppose the genotype is encoded by a sequence of length L , and the number of mismatches with respect to the optimal genotype is denoted by k . Robustness is then quantified by the maximum number of mismatches k_0 , that can be tolerated before the fitness of the individual falls significantly below that of the optimal genotype at $k = 0$.

In this contribution [1] we consider deterministic mutation-selection models of quasispecies type, which describe the dynamics of large (effectively infinite) populations [2]. We analyse the stationary state of mutation-selection balance, focusing on the dependence of the population fitness on the parameter k_0 . This allows us to identify the conditions under which a broad fitness peak of relatively low selective advantage outcompetes a higher but narrower peak, a phenomenon that has been referred to as the *survival of the flattest* [3]. Our central aim is to obtain analytic results for the robustness effect that become exact in the limit of long sequences. In particular, we clarify whether the selective advantage is a function primarily of the *relative* number of tolerable mismatches $x_0 = k_0/L$, or of the total number of mismatches k_0 .

We base our work on two complementary analytic approaches. First, recent progress in the theory of mutation-selection models [4] provides an expression for the population fitness in terms of a maximum principle that becomes exact when the limit $L \rightarrow \infty$ is performed keeping x_0 fixed. Second, Gerland and Hwa [5] have used a continuum approximation to map the mutation-selection problem onto a one-dimensional quantum problem which is then analyzed with standard techniques.

Our work was initially motivated by the observation of a discrepancy between the two approaches: Whereas the maximum principle predicts that the selective advantage of a broad peak should vanish when the limit $L \rightarrow \infty$ is taken at fixed k_0 , the quantum approach predicts that a finite selective advantage is retained in this limit. We show that the quantum approach amounts to a harmonic approximation around $k = L/2$, and improve it in such a way that the results based on the maximum principle are recovered.

The mapping to a Schrödinger equation is nevertheless useful, as it allows us to derive the leading correction to the population fitness for finite sequence length L . As a consequence we find excellent agreement between the analytic predictions and numerical solutions of the discrete mutation-selection equations (Figure 5.1).

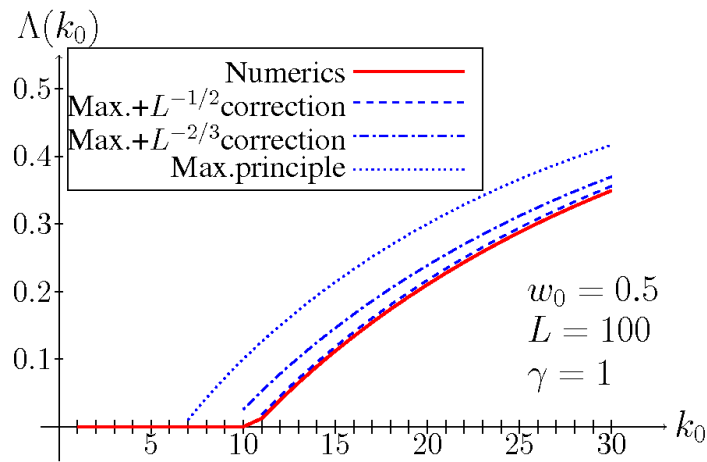


Fig. 5.1: Mean population fitness as a function of the robustness parameter k_0 for sequence length $L = 100$. The fitness advantage of a genotype with up to k_0 mismatches is $w_0 = 0.5$, and the sequence-wide mutation rate is $\gamma = 1$. For $k_0 \leq 10$ the population is delocalized, for $k_0 > 10$ it is localized around the optimal genotype. The numerical data are compared to various analytic approximations.

References

1. A. Wolff and J. Krug, Robustness and epistasis in mutation-selection models, Phys. Biol. (in press).
2. K. Jain and J. Krug, Adaptation in simple and complex fitness landscapes, in: *Structural approaches to sequence evolution: Molecules, networks and populations*, ed. by U. Bastolla, M. Porto, H. E. Roman and M. Vendruscolo, Springer, Berlin 2007, pp.299-339.
3. C.O. Wilke, J.L. Wang, C.Ofria, R.E. Lenski and C. Adami, Evolution of digital organisms at high mutation rates leads to survival of the flattest, Nature **412**, 331 (2001).
4. J. Hermisson, O. Redner, H. Wagner and E. Baake, Mutation-selection balance: Ancestry, load, and maximum principle, Theor. Pop. Biol. **62**, 9 (2002).
5. U. Gerland and T. Hwa, On the selection and evolution of regulatory DNA motifs, J. Mol. Evol. **55**, 386 (2002).

Bivariate time periodic Fokker-Planck model for freeway traffic

F. Lenz, A. Riegert, D. Herde, and H. Kantz

Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Str. 38,
D-01187 Dresden, Germany

Complex dynamical behaviour with irregular fluctuations is very often generated by the interaction of two mechanisms: Nonlinear feedback loops usually destroy the validity of simple superposition principles and also cause nonlinear input-output relations. Irregular driving forces can best be assumed to be noise which acts as additional input to the system. In such a general situation, the best modeling approach is a nonlinear stochastic model itself.

In continuous time, the most suitable model class is that of Langevin equations and the equivalent Fokker Planck equations, since here forces can be decomposed into deterministic components and potentially state dependent stochastic components. Friedrich and Peinke [1,2] demonstrated in a sequence of pioneering papers that one can indeed extract the drift and diffusion coefficients of univariate and bivariate Fokker Planck equations from experimental time series data.

Highway traffic displays strong irregular, stochastic-looking fluctuations plus evident nonlinear feedback loops with very strong deterministic components. In cellular automata models [4], e.g., deterministic rules for the velocity increment of a individual car as a function of the car in front of it together with stochastic fluctuations mimic real traffic quite well. Hydrodynamic models [5] describe traffic in a continuum approximation and hence contain exclusively deterministic feedback.

We base our model on data recorded at a single induction loop station of the highway ring around the German city of Cologne. Inspired by the work of Kriso et al. [3] we model these data by a Fokker-Planck equation in two dimensions, where our variables are the density of cars measured in car/kilometer and the flux of cars measured in cars/minute.

Kriso et al. show a vector field for the drift field in two dimensions, where clearly two rest states corresponding to the free flow and the congested flow can be discerned. However, as everyone can experience quite easily, traffic is a phenomenon with a very pronounced diurnal cycle. Traffic during the night differs strongly from the rush hours. Therefore we extend the analysis of [3] to a Fokker Planck equation with time dependent coefficients, which are assumed to be periodic in time with a period of 24 hours. When fitting drift and diffusion coefficients, we exclude all data taken from weekends or holidays, so that the weekly cycle is strongly suppressed.

In order to analyze the effect of the different times of the day on the dynamics of traffic flows we now extend our model by time dependent Fokker-Planck coefficients. We assume a separation of timescales of the fast dynamics of car density and average car velocity which is governed by a Fokker-Planck equation at each time of day and the slow periodic change of the Fokker-Planck

coefficients during the day. This approximation can be seen as a model which consists of a family of Fokker-Planck equations indexed over the times of day.

We observe that there always exists a stable fixed point at high velocities and low densities, corresponding to free flow traffic. At rush hours there is also a second stable fixed point at low velocities and high densities corresponding to congested traffic states. At rush hours our drift coefficients are qualitatively the same as the drift coefficients estimated without a resolution of the time of day in [3].

Considering only the deterministic dynamics of the system, contraction on a center manifold is observed before approaching the fixed points. Using an iterative algorithm to approximate it for different times of the day, we see that it is approximately constant over time. In terms of the microscopic traffic models [4], it corresponds to the Optimal Velocity function, determining the desired speed of a driver for a given headway.

Reduction of the dynamics onto this manifold can be attempted using the method of adiabatic elimination, which creates in first order the same results as fitting a one-dimensional Fokker-Planck equation only in the velocity but clearly differs from the transition rates in the two-dimensional model.

In summary, fitting a two-dimensional time periodic Fokker Planck equation leads to a valid model for the traffic dynamics on normal working days. It does not only reproduce the statistical features of the observed data, but supplies insight into details which would otherwise be inaccessible. Thus we can learn about the bifurcation of the vectorfield, the transition between the two stable states, but also about the approximations resulting from a one dimensional modelling approach.

References

1. S. Siegert, R. Friedrich, J. Peinke, *Phys. Lett. A* 243, 275 (1998).
2. R. Friedrich, S. Siegert, J. Peinke, et al., *Phys. Lett. A* 271 (3): 217-222 (2000).
3. S. Kriso, J. Peinke, R. Friedrich, et al., *Phys. Lett. A* 299 (2-3): 287-291 (2002).
4. K. Nagel, P. Wagner, R. Woesler, *Operations Research* 51 (5): 681-710 (2003).
5. N. Bellomo, M. Delitalia, V. Coscia, *Math. Methods and Methods* 12 (12): 1801-1843 (2002).

Very noisy Fisher waves

O.H. Hallatschek¹ and K.S. Korolev²

¹ Biological Physics and Evolutionary Dynamics
MPI DS, 37073 Göttingen, Germany

² Department of Physics, Harvard University, 02138 Cambridge, USA

The Fisher-Kolmogorov-Petrovsky-Piscounov equation is a standard model for travelling solitary waves since 70 years. Yet, the role of noise due to the discreteness of particles is still not well understood. Here, we report the effect of strong number fluctuations on the shape and speed of travelling waves in this classic reaction diffusion system. Instead of smooth sigmoidal wave profiles, we find widespread transition zones comprised of sharp fronts with power-law distributed interstices. The scaling of the wave speed depends on dimensionality, in contrast to the deterministic limit. Our analytical results agree with simulations in one and two dimensions.

Crime avalanches maybe the effect of unpunished crimes and also of jail “contamination”

J.R. Iglesias

Instituto de Física UFRGS
91501-970 Porto Alegre Brazil

We model a system of interacting agents characterized by a given wealth and a certain criminal propensity, measured by an honesty coefficient. This honesty is related to intrinsic factors, like moral barriers, and extrinsic ones, as the risk of being imprisoned if committing an offense. In the simulation, the honesty level of the agents is variable and a function of the level of punishment, on one hand, and on the contact with other agents (learning or contagion effect in prison) on the other hand. The number of crimes per habitant is measured as a function of the probability of being caught. A sharp phase transition is observed as a function of the probability of punishment. That means that once criminality has attained a high level, the probability of retribution must considerably increase in order to come back to a state of low criminality. We also analyze other consequences of criminality as the growth of the economy, the inequality in the wealth distribution (the Gini coefficient) and other relevant quantities under different scenarios of criminal activity and probabilities of apprehension.